Fixpoint Semantics for Recursive SHACL

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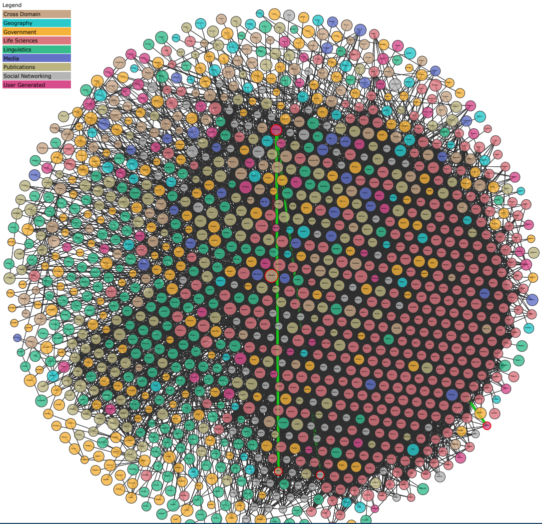
Semantic Web



Large datasets

Data quality: constraints

Shapes Constraint Language (SHACL)

SHACL [Corman 2018]

Shape expressions

$$E ::= p \mid p^{-} \mid E \cup E \mid E \circ E \mid E^{*} \mid E?$$

$$\phi ::= \top \mid s \mid \{c\} \mid \phi \wedge \phi \mid \phi \vee \phi \mid \neg\phi \mid \forall E.\phi \mid \geq_n E.\phi \mid eq(p, E) \mid disj(p, E) \mid closed(Q)$$

Schema

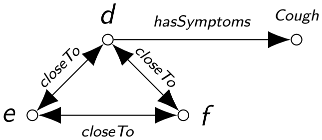
- Shape definitions: $s \leftarrow \phi$
- Target inclusions: $\phi \subseteq s$

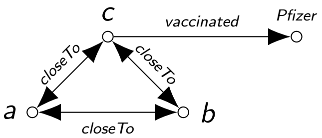
Example:

$$Isolated \leftarrow \leq_2 closeTo.\top$$

$$\exists type.\{Person\} \subseteq Isolated$$

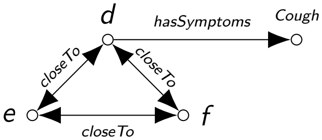
SHACL examples

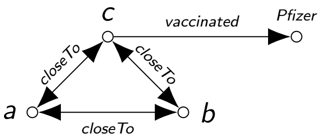




$$AtRisk \leftarrow \neg\exists vaccinated.\top \wedge \exists hasSymptoms.\top$$

SHACL examples





$$AtRisk \leftarrow \neg\exists vaccinated.\top \wedge (\exists hasSymptoms.\top \vee \exists closeTo.AtRisk)$$

Recursive Semantics: related work

[Corman 2018] — supported model semantics

[Andreșel 2020] — stable model semantics

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. . . what about other semantics?

Approximation Fixpoint Theory

Given

Complete lattice $\langle L, \leq \rangle$

(Approximation) bilattice $\langle L^2, \leq, \leq_p \rangle$

Lattice operator $O : L \to L$,

Approximating bilattice operator $A : L^2 \to L^2$:

- $O(x) \in A(x, x)$
- $\leq_p$*-monotone*

Often assumed

Symmetric: $A(x, y) = (A(y, x)_2, A(y, x)_1)$

Exact: $A(x, x) = (O(x), O(x))$

Stable operator

$S_A(x, y) \mapsto (\mathit{lfp}\, A(\cdot, y)_1, \mathit{lfp}\, A(x, \cdot)_2)$

Fixpoints

Supported: $O(x) = x$

Partial supported $A(x, y) = (x, y)$

Partial stable $S_A(x, y) = (x, y)$

Stable: x *s.t.* (x, x) *is partial stable*

Kripke-Kleene: $\mathit{lfp}_{\leq_p} A$

Well-founded: $\mathit{lfp}_{\leq_p} S_A$

Grounded: x *s.t.* $\forall v : O(x \wedge v) \leq v \Rightarrow x \leq v$.

Recursive Semantics: comparison

[Corman 2018] — supported model semantics (CRS)

Theorem

The CRS-operator is a consistent approximator.

Theorem

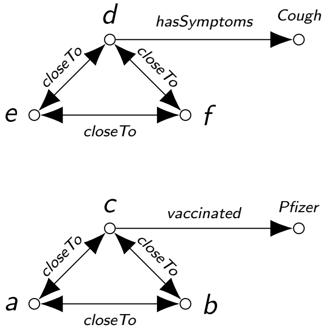
CRS-supported models coincide with AFT-supported models.

[Andreșel 2020] — stable model semantics (ACROSS)

Theorem

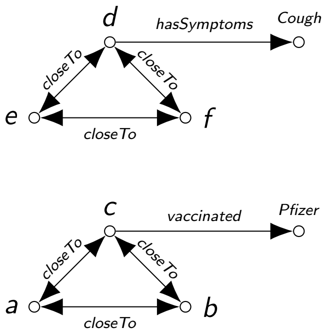
An AFT-stable model of a set of shape definitions is also an ACROSS-stable model of those definitions. If the shapes are in normal form, the converse also holds.

Difference in semantics



$$Safe \leftarrow \exists vaccinated.\top \vee \leq_1 closeTo.\neg Safe$$

Difference in semantics



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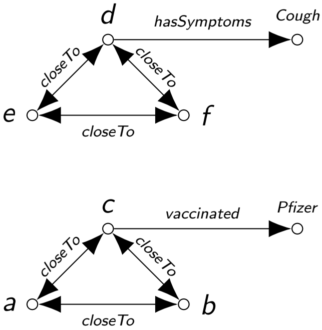
AFT-stable model
$M = \{Safe(a), Safe(b), Safe(c)\}$

Difference in semantics



$$Safe \leftarrow \exists vaccinated.\top \vee \leq_1 closeTo.\neg Safe$$

AFT-stable model
$M = \{Safe(a), Safe(b), Safe(c)\}$

ACROSS-stable models
M and $M \cup \{Safe(d), Safe(e), Safe(f)\}$

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- Semantics behave as expected

- We do not want to reinvent semantics
- We establish a strong formal foundation for the study of recursive SHACL